Variation in First Year College Students' Understanding on Their Conceptions of and Approaches to Solving Mathematical Problems

Parmjit Singh
Universiti Teknologi MARA (UiTM), Shah Alam

ABSTRACT

A primary goal of mathematics learning and teaching in secondary school is to develop student's ability to solve a wide variety of complex mathematical problems as a preparatory stage for college. In view of this, the objective of this paper is to investigate the mastery of content level that first year college students bring with them to the mathematics classroom with reference to their national examination grades (Sijil Pelajaran Malaysia). The study investigated the conceptions of 127 student and their heuristic actions in mathematical problem solving. Among these students 98.5% had obtained an A grade in the national examination for mathematics. Both quantitative and qualitative approaches, namely written assessment (five mathematical problems) and interviews were utilized in assessing student's relational understanding of mathematical concepts in problem solving. The percentage of correct responses from these students, who can be considered as the cream of the crop of the nation, for the five items in the written test were 74.8%, 72.4%, 67.7%, 52.8% and 18.9 %.The data indicates that the grades obtained in the national examination did not reflect their mathematical knowledge in mathematical problem solving. The study concludes that capable mathematics students (the ones who obtained an A grade in the national examination) when placed in the context of non-routine problems have difficulty solving what may be *considered elementary mathematics for their level of task achievement. This evidence seemingly indicates that many students from high schools are not acquiring the mathematical skills expected of college level mathematics.*

***Keywords**: problem solving, proportion, ratio, mathematics, learning, school mathematics.*

Background

Students coming to college need to unpack and revisit their mathematical knowledge which they bring from school to allow them to examine the undergirdings and interconnections of college mathematics with other areas of mathematical application such as physics and chemistry (Parmjit & White, 2006). These students with a string of A's in their bag do indeed construct a reasonably large number and a variety of algorithms in order to continue achieving good results in mathematics examinations. However, it is of interest to determine the quality of this knowledge since the quality of students' mathematics knowledge is always a crucial matter. The single most significant factor determining the quality of knowing is the quality of the students' experiences in constructing their knowledge. The use of problem solving in the college mathematics classroom content enables this unpacking and can result in variation in these students' conceptions of and approaches to solving fundamental mathematical problems.

Many teachers, parents, students, and even educators equate problem solving with word "problems" that are presented in textbooks or in various level of examinations. But are these "problems" really "the" problems? Before coming into details of this, we do need to define what a problem is in mathematical contexts. Reitman (1965) defined a problem as when you have been given the description of something but do not yet have anything that satisfies that description. His discussion described a problem solver as a person perceiving and accepting a goal without an immediate mean of reaching the goal. In a problem, one is not aware of any algorithm that will guarantee a solution. As Polya (1973) puts it:

> *To have a problem means to search consciously for some action appropriate to attain some clearly conceived but not immediately attainable aim. To solve a problem means to find such an action* (p. 99).

Wheatley (1991) also succinctly pointed out that solving a problem means finding a way out of difficulty, a way around an obstacle, attaining an aim that was not immediately attainable. In a parallel note, Reys et al., (2004) defined a problem as "a situation in which a person does not know immediately what to do to get it" (p. 115). Reys et al. (2004) believed that the difficulty of a problem must require "some creative effort and higher-level thinking" (p. 115) to resolve. Schoenfeld (1985) also emphasised that the "difficulty should be an intellectual impasse rather than a computational one" (p. 74). In summary, one can conclude that a question is a problem if the procedure or method of solution is not immediately known, and hence requires one to apply the previous constructed knowledge in a new and unfamiliar situation with the aid of creativity.

Problem solving has a special importance in the study of mathematics. A primary goal of mathematics teaching and learning is to develop the ability to solve a wide variety of complex mathematical problems. This was also noted by Garfola & Lester (1985):

> *The primary purpose of mathematical problem solving instruction is not to equip students with a collection of skill and processes, but rather to enable them to think for themselves.* (p. 166)

Teaching college students how to use mathematics to reason, to think critically and to solve problems is a key to the success of any mathematics curriculum and it has long been an issue of concern at every level. Yes, one will not deny that successful mathematics students do indeed construct a fairly large number and variety of algorithms in order to continue achieving good results in national mathematics examinations. However, what is the quality of this mathematical knowledge? A variety of research has been done in Malaysia in assessing student's relational understanding of mathematics. Relational understanding, which is at times synonymously used as conceptual understanding, relates to employing reasoning "to what and the why" rather than "what to do to get the answer". In two separate studies (Parmjit, 2000, Parmjit & White, 2006) to determine the relationship between secondary (upper and lower) students' grades obtained in national examinations and their mathematical knowledge, it was found that the grades obtained did not reflect their mathematical knowledge in mathematical problem solving. The conclusion seems to indicate that students in Malaysia have learnt how to do numerical computation at the expense of learning how to think and solve problems. This is because the emphasis in the examinations has been on solving routine problems rather than non- routine problems. The ineffective practices that are prevalent in today's classroom are: teachers expect students to learn mathematics by listening and imitating; teachers teach as they were taught rather than as they were taught how to teach; teachers teach only what is in textbooks; students learn only what will be on the test. Although in the Malaysian Mathematics Education syllabus, problem solving has been stated as a theme, these problems are of the routine type rather than the non-routine type. What is the difference between these two problems? Let's discuss this in the context of problems available in Malaysian textbooks.

> For example, Task 1: There are 15 people at a gathering and each of them shake hands once and only once with everyone else. How many handshakes are there altogether?

This task may be familiar to students who have learnt it in high school and so they may know immediately how to solve it. The routine procedural skill that students will employ is $15C_2$ which produces 105 handshakes. This task is considered as routine for maths students because of their familiarity with it. One might question that this task might be a problem for students who are low achievers as they might not know how to apply it properly. However, with enough practice, this task can become a routine exercise for these students. Moreover, the purpose of this type of tasks is to "provide students with practice in using standard mathematical procedures (for e.g., computational algorithms, algebraic manipulations, and use of formulas", Lester, 1980, p. 31). Some researchers called this type of tasks "routine problems" (Orton & Frobisher, 1996, p. 27).

Let us contrast task 1 with another task.

> Task 2: How many "zeros" are there in 100?

This task is inherently different from the first task in that it requires some higher level thinking strategies (e.g Reys et. al., 2004) and not just a direct application of a procedure. Furthermore, it cannot be computed by using a scientific calculator because of the number of digits involved. The first type of mathematical tasks (Task 1) is called "routine problem" since that is commonly found in textbooks and involve practicing procedures, and the second type (Task 2) "non-routine problem" is not commonly found in texts and requires the use of higher level thinking strategies to solve.

Any institution that wishes to retain basic mathematics students cannot simply hustle them into the regular curriculum and assume that a few hours of tutoring will enable them to learn the material. Neither can the institution dredge up a standard high school course to offer them. Rather, the institution must assess both the mathematical content that is required for college – level courses and the mathematical skills of the incoming students. Only then can a curriculum be designed which will best help these students to face the challenge of college mathematics. In Singapore, mathematical problem solving is central to mathematics learning at both primary and secondary levels. It involves the acquisition and application of mathematics concepts and skills in a wide range of situation, including non-routine, open ended and real word problems (Lee, 2006).

Another issue of concern is the methodological approaches employed in Malaysia, especially in assessing college students' mathematics learning. Too often, we consider only aspects of knowledge that focus their attention upon the concepts identified, the generalization recalled, the problem solved, the theorem proven or the procedure extended. The emphasis is upon the mathematical result, rather than the process of constructing the idea. As educators, we would like to see students develop their reasoning and thinking capabilities rather than their abilities to memorize meaningless facts. A particular fruitful approach for research in developing these capabilities is to concentrate on the students themselves and the ways in which they individually construct knowledge rather than just solely rely on what they can and cannot do. We as educators should be critical of the quality of research in mathematics education in colleges. One can look at tables of statistical data utilizing powerful quantitative analysis and can say "so what!" Vital questions go unanswered while means, standard deviation, and t-tests pile up. There is too great a reliance on statistics, and a deep look at process is avoided. Statistics are valuable in their place. They can suggest hypotheses in preliminary studies and help to test them in well-designed experimental studies. But if we want to understand what goes on in students' heads when they solve problems, we have to watch them solving problems (Schoenfeld, 1987).

Students' misconceptions can remain hidden from the lecturer's view unless particular attention is paid to the way they think through a problem. For this study, these students have been through the national standardized examination, namely the Malaysian Education Certificate (SPM), and thus the researchers could consequently pursue the question of how well Malaysian Education Certificate mathematics grades reflect the students' conceptual understanding of mathematics. It is believed that only by closely scrutinizing students' thinking during a problem session can lecturers reveal this sort of misconception.

Objectives of the Study

This paper assesses students' basic mathematical knowledge upon entering college. The general objective is to investigate the mastery of content level which first year college students (fresh from high school) bring with them to the mathematics classroom based on their national examination grades (Sijil Pelajaran Malaysia). It investigates students' conceptual understanding and approaches in solving non-routine mathematical problems, identifies what sort of experiences and understanding are critical in solving the given tasks and the difficulties they encounter. The goals are primarily to make sense of students' mathematical behavior – to explain what goes on in their heads as they engage in mathematical problem solving tasks of some complexity.

Methodology

The methodology utilized in this study encompasses quantitative and qualitative method with a greater emphasis on the latter. The study investigated 127 first year college students' conceptions and heuristic actions in mathematical problem solving.

The instrument used for this study was adapted from Parmjit (2006) and four items were elicited from that study. The difference between the study conducted in 2006 and this study is that the former emphasize quantitative data analysis while the latter emphasizes qualitative data analysis. Second, the students used in the study in 2006 were first year students who were at the end of their semester while the students in this study are randomly selected in their first week of college when they registered for a problem solving course.

There were five items in this test and the responses were grouped into categories according to the criterion behaviour exhibited. A numerical value was assigned to each of these criterion behaviours. Students' responses were categorized on the following five point scale based on the reasoning employed:

4. All correct, good reasoning
3. Good reasoning, small error(s)
2. Some promising (reasoning) work but it is not clear on whether a solution would be reached
1. Some work but unlikely to lead to a solution
0. Blank

However, for the quantitative responses of this paper, the analysis was computed based on correct and incorrect response. The scale of 0, 1 and 2 were categorized as incorrect responses and 3 and 4 as correct responses.

The content validity of this instrument was established by three experienced mathematics lecturers who were experts in the related area. The content was validated based on the specification of KBSM mathematics syllabus. Several suggestions were given and the items were amended accordingly. In determining the reliability estimation, test-retest reliability estimation was utilized. This estimation was based on the correlation between two administrations of the same test to a group of pupils. The researcher was aware of the recall biasness (exposure to the test at Test 1 influences scores on the test at Test 2) associated with the test-retest reliability analysis and in view of this, the second test was conducted three days after the first and the items in the retest was randomly arranged. Field (2005) considered as good reliability coefficients ranging between 0.70 and 0.80. The test-retest reliability coefficient of 0.917 is a strong reliability coefficient. In other words, the correlation coefficients are of substantial magnitude indicating high stability of test results over time and they provide good evidence of the reliability for the instrument.

One hundred and twenty seven first year college students, aged 18-19, participated in this study. Of these, 77.2% obtained a grade 1A in their national examination as compared to 21.3% and 1.5% who obtained a grade 2A and 3B respectively (Table1). In other words, 98.5% of the students involved in this study obtained an A grade in the national examination. The rationale for choosing this sample was that they have at this level been formally taught in secondary school the basic mathematical concepts needed to solve problems.

For the qualitative analysis, eight students were selected based on the responses given in the written test and each interview session lasted

Table 1: Distribution of Samples According to Mathematics Grades

Grade	Frequency	Percentage
1A	98	77.2
2A	27	21.3
3B	2	1.5
Total	127	100.0

for about an hour. The composition of the students according to their grades are shown in Table 1.

Findings and Discussion

The following sections detail the data obtained from both the written test and interviews from the five items used in this study.

Table 2 shows 74.8% of the students in this study got item 1 correct as opposed to 25.2% who got it incorrect. The data also indicates that 32 (25.2%) students of the total who obtained an incorrect response utilized the additive reasoning procedures in deriving the answer.

Table 2: Students' Responses for Item 1 (n = 127)

	% Correct	% Incorrect
Pele and Maradona were great soccer players. Pele scored 300 goals in 400 matches, while Maradona scored 400 goals in 500 matches. Who had a better scoring record: Pele, Maradona or they have a same scoring record? (Please explain).	74.8	25.2*

*32 students gave an incorrect response due to additive reasoning

Among the common reasoning for incorrect answers given based on students' worksheet are as follow;

- The difference between the goals are the same (400 – 300 = 100 and 500 – 400 = 100), so both Pele and Maradona have the same scoring record.
- Some of them computed the ratio (400/300 for Pele and 500/400 for Maradona) and obtained 1.33 and 1.25 respectively. Then they

reasoned that since the value of the former is greater than the latter, they made the conclusion that Pele has a better scoring record. Surprisingly, they did not seem to know what the number represents.

- The difference is 100, so the scoring record is the same.

Interview with students confirmed the reasoning made above. Student AS_A stands for students A, S_B for students B and etc.

S_A : *I think both are the same... the goal difference is one hundred.*
R : *How did you get one hundred?*
S_A : *Four hundred minus three hundred is one hundred and also five hundred minus four*
R : *How do you interpret this one hundred?*
S_A : *The goals that they missed.*

From item 1 as shown in Table 2, 32 students or 25.2% of the students obtained an incorrect response which was based on additive reasoning where the answer given was " both have the same scoring record". Furthermore, these students were not able to conceptualize and interpret the ratios computed (400/300 for Pele and 500/400 for Maradona).

In a follow up question;

R : *If there is another player who has played 600 games, how many goals must he score in order to have the same scoring record as Pele?*
S_A : *He needs to score five hundred (500) goals.*

This indicates the consistency in student A reasoning based on additive reasoning. He was not able to see and compare the ratio of the goals to the number of games played.

The heuristics performed by student A was as follows:

Pele: 400 – 300 = 100 goals
Maradona: 500 – 400 = 100 goals

He was unable to coordinate two ratios simultaneously (400/300 and 500/400). He surmised that since the differences between the two players are the same, they will have the same scoring record!

Student B

S_B : *Pele has a better scoring record.*
R : *Why?*
S_B : *(Pointing to his work sheet) because four hundred over three hundred (400/300) for Pele...and you get 1.333 and five*

hundred over four hundred (500/400) for Maradona...you get 1.25

R : *So, what does these values mean (referring to 1.333 and 1.25)?*

S_B : *Pele's value is bigger (pointing to 1.333), so his scoring record is better.*

There were a number of students who were able to use proportionality (coordination of two ratios) but from the interview, one of them showed that he is unable to compare or relate the obtained ratios. This is noticeably seen in the transcripts below:

Student C (who obtained the correct answer)

S_C : *Maradona has a better scoring record.*

R : *Why?*

S_C : *(pointing to her worksheet) three hundred over four hundred (300/400)…0.75 and four hundred over five hundred (400/500)…0.8.*

R : *What is represented by 0.75?*

S_C : *Goals in one game.*

R : *What about 0.8?*

S_C : *For Maradona.*

R : *What about comparing the numbers 0.75 and 0.8?*

S_C : *Actually the distance between 0.75 and 0.8 is very small if the difference is 100. If he plays 100 games, the difference is only 0.05.*

These students were unable to coordinate the information given where one needs to compare the ratio of the goals to the number of games played (or the two scoring rates). The mere computation manipulation does seem not to make much sense to these students, especially when the numbers did not divide evenly (e.g. 400/300 = 1.333...). Here, the students did not have a good notion of ratio nor did they have the understanding that the objective of a proportion problem is to keep the value of a ratio invariant under iteration. Thus, apparently for these students, it seemingly indicates that when determining whether or not two ratios were equivalent, they looked for a relation between the first rate pair and then matched it with the second rate pair, to see if the relation holds. However, the conceptions of the ratios (e.g. 0.75 or 0.8) confused these students' interpretations. These difficulties are often attributed to the insufficient integration of instruction with student's intuitive knowledge about quantities as they experience in everyday activities.

Approximately 27.6% of the students obtained an incorrect response for this item (item 2 in Table 3), which is quite similar to item 1 as shown in Table 2. From this percentage, 33 students or approximately 26% utilized the additive reasoning procedure for their rationale. They reasoned that if they add one can of black paint and one can of white paint, their colour should still remain the same. Their reasoning was based on the assumption that since one can of each paint was added and the quantity of both colours was the same, the new mixture should have the same shade. Their reasoning was based on the primitive additive reasoning and approximately 26% of the students gave this additive reasoning. In short, these students failed to construct a coordination of two ratios simultaneously as: 2 white to 3 black and 3 white to 4 black.

Table 3: Students responses for Item 2 (n = 127)

	% Correct	% Incorrect
Eva and Alex want to paint the door of their garage. They first mix 2 cans of white paint and 3 cans of black paint to get a particular shade of gray. They add one more can of each. Will the new shade of gray be lighter, darker or are they the same?	72.4(92)	27.6*

*33 students used addition reasoning

Responses from the interview with students who obtained the correct response show that they had the idea as to how to solve the problem. The researcher needed to ask the right questions to perturb the students. The student then used proportional reasoning and proceeded to use multiplicative reasoning stating that in order to get the same shade, the ratio must be 4/6. They were then able to see that 2/3 and 3/4 represented the ratios of white paint to black paint and the greater the number, the lighter the shade of the mixture.

S_E : *I got the ratio of 2/3 and 3/4.*
R : *How do you use this ratio in determining which shade is lighter, darker or the same?*
S_E : *.... to be same, the ratio should be equal.*
R : *So, if 2/3, we compared it with?*
S_E : *4/6*
R : *what about 2/3 and 3/4?*
S_E : *We look at the paint. If there is more black paint, then it is darker.*

The following verbatim is based on students who used the percentage method with incorrect responses (based on their answer sheet). On their earlier attempt, they tried utilizing proportional reasoning but were unable to conceptualize the meaning of the percentages of 66.7% (2/3) and 75%. (3/4)

R : *You obtained this figures, 66.7 % and 75 %. And then you said that the shade is darker. Why?*
S_F : *It is the same.*
R : *Same?*
S_E : *Same shade.*
R : *Why, now you say it is the same?*
S_E : *Because they added one can of white and one can of black…the same number of cans for each.*
R : *What about the percentages you computed?*
S_E : *It is wrong.*
R : *Why? (Could not give an explanation)*

Students (e.g. S_E) could not relate the percentage to the problem given. This led them to use additive reasoning to get a similar shade. They then concluded that an equal amount of each type of paint is added to the mixture; therefore, the mixture has the same shade.

A third group of students also used additive reasoning. The reasoning employed is that if an equal number of cans for each type of paint is added to the mixture, the shade will remain the same. They were unable to see the proportion of white paint to the black paint before and after the addition of two cans of paint.

S_G : *In my opinion, it will be the same as one tin of white paint and one tin of black paint is added... because the difference is the same. If we want a darker shade, we will have to add more black paint than white paint... and if we want a lighter shade, more white paint than black paint.*
R : *So, in your opinion, if one more tin of white paint and one more tin of black paint is added, the shade of gray is the same?*
S_C : *Yes. (very confidently)*

In both these items, (item 1 and item 2), what it requires is making sense of the structural similarity between a/b and c/d based on the invariance of ratio. The comparison problem of proportionality, the relationship among a, b, c and d are understood as two ratios or rates, i.e. a/b and c/d. These ratios (rates) are stored and further compared to determine whether or not they are equivalent and /or whether another level of relationship is in question.

Next, Table 4 indicates 67.7% of the students arrived at the correct answer for item 3. 32.3 % of the students failed to see an inverse proportion relationship and solved the question by utilizing a cross multiplicative strategy. Many of them used the following heuristics:

9 workers = 5 hours
6 workers = X
X/5 = 6/9; 9X = 30; x = 30/9 = 3 1/3 hours.

Table 4: Students' Responses for Item 3 (n = 127)

	% Correct	% Incorrect
If it takes 9 workers 5 hours to mow a certain lawn, how long would it take 6 workers to mow the same lawn? (Assuming that the workers are all performing at the same rate and all working for the entire time).	67.7(86)	32.3*

*31.5% used direct proportion

In fact, 31.5% of the students utilized mechanical reasoning as shown above.

Here, they did not reason what each number represents and what they were actually computing. Logically, they should have realized that the answer they produced (3 1/3 hours) implied that fewer people take a shorter time to finish up the job!

An interview with a student who used a similar method revealed that he was aware that fewer workers means longer working hours but was unable to answer why his cross multiplication strategy did not give a logical solution.

R : *What is your answer?*
S_H : *Three hours and thirty three minutes (3 hours 33 minute)s.*
R : *How did you obtain this answer?*
S_H : *Well (pointing to his worksheet), nine workers takes 5 five hours....we need to find six workers ....so, six over nine multiply with five (6/9 x 5)...the answer is ten over three (10/3)...equals to three-one over three (3 1/3).*

R : *What is the unit for three one over three?*
S_H : *Hours...it is three hours and .....twenty minutes (3 hours 20 minutes).*
R : *Is your answer logical?*
S_H : *...kept quiet*
R : *What are you thinking?*
S_H : *If nine workers takes five hours, then six workers should be longer!*
R : *What do you mean by longer?*
S_H : *Time*
R : *So, where is your error?*
S_H : *My working is correct (showing his steps, utilizing the cross multiplication method, in his worksheet)… I am not sure.*

An interview with another student yielded a similar response.

R : *In here (pointing to the worksheet) you wrote five hours multiply with 60..then divide by nine. Why?*
S_C : *To find how many minutes each worker takes. Then multiply with six (workers).*
R : *So, how long does it take for six workers?*
S_C : *3.33 hours*
R : *Are you happy with your answer?*
S_C : *Yes*
R : *If nine workers take five hours, fewer workers will take shorter or longer time?*
S_C : *(After a while)....something not right.*
R : *What do you mean?*
S_C : *Should take a longer time because fewer workers should take longer time!*

After realizing the mistake, he was able to solve the problems and derived the answer as seven hours and thirty minutes (7 hours 30 minutes.)

The most common incorrect answer to this problem was due to the use of an inappropriate direct proportion formula and the failure to realize that the resulting answer was unreasonable. Approximately one-third (32.3%) of these students, who obtained an A grade in the national examination were unable to solve this problem. For these students, the word proportion seemed to be equated with direct proportion though the inverse proportion content has been learnt in high school. The data from these problems seems to indicate that school mathematics instruction was procedural without sense making: one learns to read the problem, extract the relevant numbers and the operation to be used, perform the operation and write down the result–without even probing into what it all means.

The majority of the those student who used proportional reasoning by cross multiplication, multiplication by 6 to get the total numbers of hours work by 6 workers, did not realize that their solution was illogical. Utilizing this approach simply becomes an act of symbolic manipulation. This cross multiplication algorithm is an efficient way of getting answers but is often used in an absurd way. The premature formalism leads to symbolic manipulation which students cannot connect to the real world, resulting in the virtual elimination of any possibility in enhancing their thinking capabilities. As Kieran (1988) pointed out, "symbolic knowledge that is not based on understanding is highly dependent on memory and subject to deterioration" (p. 178).

Table 5: Students' Responses for Item 4

	% Correct	% Incorrect
An old antique bicycle has wheels of unequal size. The front wheel has a circumference of 8 feet. The back wheel has a circumference of 10 feet. How far has the bicycle gone when the front wheel has turned 20 more revolutions than the back wheel?	52.8 (67)	47.2*

*32.5% without reasoning

Table 5 shows that only 52.8 students obtained a correct solution for item 4. Observations of students' responses to this item indicate that 32.5% (those with an incorrect solution) of the students could not even reach a stage to show any promising heuristic reasoning. They faced great difficulties in expressing the problem into a mathematical expression. The interview results revealed that a majority of them did not realize that both tyres travelled the same distance. Some of the heuristics by students who got it correct are as follows:

The majority of students who obtained a correct response used the following algebraic method:

x - number of revolution
Front wheel revolution: 20 + x
Back wheel revolution: 10x
Distance travelled by front wheel = 8 (20 + x) = 160 + 8x
Distance travelled by back wheel = 10x
Since the distance travelled is the same,
160 + 8x = 10x
x = 80
Bicycle travelled for a distance of 10 (80) = 800 feet

Another group of students utilized proportionality to solve the problem.

To travel a certain distance, the front wheel has made 5 revolutions, while the back has made 4. Therefore, the ratio is 5: 4, and the difference is 1 revolution. So, to get a difference of 20, multiply ten on each side to get 100: 80. This shows that the front has made 100 revolutions. Hence, the wheel has travelled 100 x 8 = 800 feet.

As shown in this item, students can no longer function optimally in mathematics learning by just knowing the rules to follow to obtain a correct answer. They also need to be able to decide through a process of logical deduction what algorithm, if any, a situation requires and sometimes, need to be able to develop their own rules in a situation where an algorithm cannot be directly applied. I believe that it is time for schools to focus their efforts on preparing people to be good adaptive learners as Resnick (1987) argued, so that they can perform effectively when situations are unpredictable and tasks demand changes, as required by item 4 in this paper. In other words, students need to learn the means by which mathematics can be applied to a variety of unfamiliar situations.

Table 6 shows that 81.1% of the students obtained an incorrect response for item 5. This was the item where students faced highest

Table 6: Students' Responses for Item 5

	% Correct	% Incorrect
A van travels a maximum of 100 km/hr. Its speed decreases in proportion with the number of passengers. The van can carry a maximum of seven people. Given that the van can travel 88km/hr with 3 people in the van, what will be the speed of the van when 6 people are on board?	18.9(24)	81.1*

*49.5 % responded 97 km/hr

percentage of difficulty as compared to the other questions. From the responses on the worksheets, it was noticed that these students were not able to model the situation in a mathematical form. Various non logical computation were given as the solution, however, from those with an incorrect response, 49.5% responded to the solution as 97 km/hr.

Interviews with students based on these responses are as follows:

Student E conceptions:

R : *How did you obtain this answer? (Pointing to her answer, 97km/hr).*

S_E : *Speed decreases when the number of people increases. Eighty eight km per hour for three passengers, so we need to find for six passengers?*

R : *How did you find this? (pointing to the table, as illustrated, in the worksheet)*

peed (km/h)	100	97	94	91	88
People	7	6	5	4	3

The steps shown on her worksheet:
For 5 people, middle = (88 + 100)/2 = 94 km/hr
For 4 people = (94+88)/2 = 91 km/hr
6 people = (94+ 100)/2 = 97 km/hr

S_E : *To find for five people, it is in the middle between eighty eight and one hundred, so eighty eight plus one hundred and divide with two... ninety four km/hr...*

R : *How do you know it is in the middle?*

S_E : *Because, five is between three (3) and seven (7) ... and four (4) is between three (3) and five (5)... I mean people*

R : *What about six people?*

S_E : *Similar, ninety four plus one hundred and divide with two... ninety seven (97) km/hr*

Student F conceptions:

S_F : *With three people, it travels 88 km/hr, with seven people... 100 km/hr*

Pointing to her worksheet (as illustrated):

3 people – 88 km/hr
2{ }6
5 people – 94 km/hr
2{ }6
7 people – 100 km/hr

S_F : *For five people, it is 94 km/hr.*
R : *How did you get the speed for five people?*
S : *The difference between 3 (people) and 7 (people) is 4 and the difference between 88 (km/hr) and 100 (km/hr) is 12 (km/hr). So, every increment in two people we can add a speed for 6 km/hr... One passenger... increment of 3 km/hr.*

From the worksheet:

3 p – 88 km/hr
4 p – 91 km/hr
5 p – 94 km/hr
6 p – (94+3) km/hr = 97 km/hr

The majority of the other students also produced a similar reasoning in their solutions.

The majority of students with correct responses used the following algorithm(using the algebra abstraction) to solve the problem.

Let say, x is the reduction in speed per person:

$100 - 3x = 88$
$100 - 88 = 3x$
$12 = 3x$
$x = 4$ km/h reduction in speed per person

When six person are on board, the van travels at
$100 - 6x = 100 - 6(4)$
$= 76$ km/h

From these interviews, it indicates that both S_E and S_F did not have a good notion of ratio nor did they have the understanding that the objective of a proportional problem is to keep the value of a ratio invariant. Thus, it seems that when determining whether or not the two rates were equivalent, they looked for a relation between the rates.

S_E looked for a relation between the first rate 88km/hr and 100 km/hr and then matched it with 94km/hr and 88 km/hr to see if the relation holds. Similarly, student F matched the relation between 3 people and 7 people and matched it with 88 km/hr and 100 km/hr. Since the reason for a search for a multiplicative relationship was not understood, they looked for any relationship where the pairs match.

Sometimes these students' failure to apply a multiplicative strategy was not due to the absence of that multiplicative strategy from their repertoire. Rather, the application of an incorrect and less sophisticated strategy was also due to lack of domain in context familiarity. That is, students failed to recognize that the situation called for ratio and proportion. For these reasons, problem solving can be developed as a valuable skill in itself by contextualizing daily life problems and as a way of thinking (NCTM, 1989), rather than just as the means to an end of finding the correct answer. Perkins (1981) concisely states that good thinkers do not necessarily think harder, longer or more exactly; they have simply learned to think in directions that are more likely to be productive.

From the interviews with students, the joy of doing these kinds of problems allows the students to experience a range of emotions associated with various stages in the solution process. They also show the willingness and desire to engage with the tasks for a longer period of time. Although it is this engagement that initially motivates the solver to pursue a problem, it is still necessary for certain techniques to be available for the involvement to continue successfully. Hence, more need to be understood about what these techniques are and how they can best be made available. Problem solving has a place in our curriculum as spelt out in our mathematics syllabus; however, it is often used in a token way as a starting point to obtain a single correct answer, usually by following a set of "correct procedures".

Conclusion

This study indicates that capable mathematics students, with an A grade in national examinations have, when removed from the context of coursework, difficulty doing what may be considered elementary mathematics for their level of achievement. As the data shows, these first year college students faced difficulty in the application of elementary mathematical concepts to the given problems.

Too often, they utilized algorithmic procedures (e.g. cross multiplication technique) that are alien to them in terms of their conceptions. These techniques may be useful for getting the answers to a problem but they do not provide rich learning opportunities modeling the situations. These students have learned how to do numerical computations at the expense of learning how to think and solve problems. One can surmise that though they are first year college with A grades in the national mathematics examination, they still do not have a good notion of the intensive value of ratio, especially as indicated in task 1, task 2 and task 5. Nor do they have a sound understanding that the objective of a proportion problem, that is to keep the value of ratio invariant. Thus, they are supposed to look for a relation between one ratio and then match it to the second ratio based on multiplicative structure. However, in this study (based on the interviews (e.g. S_A, S_B, S_C), the students look for a relation between one ratio and another but one that is based on the incorrect assumption of additive reasoning.

The relationship between students' grades obtained in national examination and their mathematical knowledge was studied by the researcher (Parmjit et. al, 2002; Parmjit, 2006) and it was found that the grades obtained did not reflect their mathematical knowledge in mathematical problem solving. This study embarked on a similar mission with the difference that whereas the latter study focused more on qualitative analysis, the former focused on quantitative analysis. The findings of this study detail a similar outcome where students' grades from the national examination did not match their content knowledge of mathematics. 98.5% of the students in this study obtained an A grade in the national examination for mathematics. However, from the five items given, the percentage of correct responses obtained from these students, who can be considered as the cream of the crop of the nation were 74.8%, 72.4%, 67.7%, 52.8% and 18.9 %. The items chosen for this study were based on the content of secondary school mathematics where the usage of calculator was not required. It focused more on conceptual development rather than computation.

Successful mathematics students do indeed construct a fairly large number and variety of algorithms in order to continue achieving good results in mathematics examinations. However, many students emerge from their study of mathematics in schools without a functional understanding of some elementary but fundamental concepts with emphasis on algorithmic procedure rather than conceptual understanding. These underlying concepts which are the basis of understanding mathematics become a secondary entity in learning and the algorithmic procedures in producing the product becomes the prime entity of learning. I am not pointing that memorizing is not good but rather that emphasis should be more on the understanding of conceptual facts. As Pirie (1988) pointed out:

> *An algorithm is not of itself knowledge, it is a tool whose use is directed by mathematical knowledge and care must be taken not to confuse evidence of understanding with the understanding itself.* (p. 4)

Evidence from a variety of sources makes it clear that many students are not learning the mathematics they need or are expected to learn (Parmjit, 2003; Kenney and Silver, 1997; Mullis et al., 1998). Studies have shown that students who score well on standardized tests often are unable to successfully use memorized facts and formulae in real-life application outside the classroom (Parmjit, 2000; Parmjit et al., 2002). Resnick (1987) has also commented that practical knowledge (common sense) and school knowledge are becoming mutually exclusive. This was echoed by Steffee (1994):

> *The current notion of school mathematics is based almost exclusively on formal mathematical procedures and concepts that, of their nature, are very remote from the conceptual world of the children who are to learn them.* (p. 5)

A review of recent studies in Malaysia (e.g. Parmjit & Lau, 2006; Parmjit & White, 2006; Fatimah, 2007; Lau, 2006) suggests that possible problems in secondary school mathematics may be due to the procedural paradigm orientation in the curriculum and the conventional style of teaching in the classroom which do not provide sufficient opportunities for students to develop conceptual understanding. The current notion of school mathematics is based almost exclusively on formal mathematical procedures and concepts that, of their nature, are very remote from the conceptual world of the students who are to learn them. Many students see little connection between what they study in the classroom and real life. Just having students memorize facts and algorithms is debilitating. "Learning mathematics involves the construction of a network of meanings-relating one thing to another" (Wheatley, 1991). While students are memorizing facts, which could not possibly hold any meaning for them, they are not constructing relationship and patterns. In fact, they may "stop thinking about mathematical relationship" altogether (Wheatley, 1991). As examination grades attest, many of these students in this study who complete high school can solve standard quantitative problems, such as those in the national examinations. Success on such problems, however, does not ensure that students have developed a functional understanding, i.e., the ability to do the reasoning needed to apply appropriate concepts and principles in situations not previously memorized. For many students, solving such problems is a relatively passive experience. Problems that require non-routine tasks or qualitative reasoning and verbal explanation demand a higher level of intellectual involvement.

Helping students develop problem solving skills is a frequently cited goal of educators. The task of providing a mathematical education in problem solving that is meaningful and relevant to students is a formidable one. As educators, we would like to see students develop their reasoning and thinking capabilities rather than their abilities to memorize meaningless facts. Emphasizing variations in students' conceptions of and approaches from this study, hopefully, can act as a catalyst in driving an initial effort towards making problem solving as the central focus in mathematics learning.

References

Field, A. P. (2005). Discovering Statistics Using SPSS (2nd Edition). London: Sage.

Garfola, J. & Lester, F. K. (1985). Metacognition, cognitive monitoring, mathematical performance. *Journal for Research in Mathematics Education, 16*, 176.

Kenney, P. A., & Silver, E. A. (1997). Results from the sixth mathematics assessment. Reston, VA: National Council of Teachers of Mathematics. Retrieved September 25, 2008, from *http://www.ascd.org/handbook/demo/math/framesection3.html*

Kieran, T. (1988). Personal knowledge of rational numbers: Its intuitive and formal development. In J. Hiebert & M. Behr (Eds.), *Acquisition of mathematics concepts and processes* (pp. 45-90). New York: Academic Press.

Lau Ngee Kiong (2006). Mathematical problem solving: Interdependence of problems, problem solving and metacognition. In Parmjit, S., and C. S. Lim (Eds.), *Improving teaching and learning of mathematics.:From research to practice* (pp. 165-198). Shah Alam: Universiti Teknologi MARA, Malaysia.

Lee Peng Yee (2006). *Teaching Secondary School Mathematics: A resource book* (p. 305-307) Singapore: McGraw Hill.

Lester, F. K., Jr. (1980). Problem solving: Is it a problem? In M. M. Lindquist (Ed.), *Selected issues in mathematics education* (pp. 29-45). Berkeley, CA: McCutchan.

Mullis, I. V. S., Martin, M., Beaton, A., Gonzalez, E., Kelly, D., & Smith, T. (1998). Mathematics and Science achievement in the final year of secondary school: IEA´s Third International Mathematics and Science Study (TIMSS). Chestnut Hill, MA: Retrieved October 25, 2005, from *http: 216.109.117.135* .

National Council of Teachers of Mathematics (NCTM) (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

Orton, A., & Frobisher, L. (1996). *Insights into teaching mathematics.* London: Cassell.

Parmjit S. (2000). Understanding the concepts of proportion and ratio among grade nine students in Malaysia. *International Journal of Mathematics Education, Science and Technology, 31* (4), 577-599.

Parmjit, S; Nuraini Yusoff & Seripah Awang Kechil (2002). An assessment of college students understanding in mathematical problem solving. In the proceedings of Seminar INSIGHTs 2002, University Technology Mara, Shah Alam, Selangor, Malaysia.

Parmjit, S. (2003). Procedural orientation of school mathematics in Malaysia. *Educators and Education Journal, 18,* 58-64.

Parmjit, S., & White, A. (2006). Unpacking first year university students' mathematical content knowledge through problem solving. *Asian Journal of University Education, 1,* 33-56.

Parmjit, S., & Lau, N. K. (2006). Mathematical problem solving: Where are we heading? *Journal of Educational Research*, *26*, 39-50.

Perkins, DN (1981). *The Mind's Best Work*. Cambridge, MA: Harvard University Press.

Pirie, S. (1988). Understanding: Instrumental, relational, intuitive, constructed, formalized? How can we know? *For the Learning of Mathematics*, *8*(3), 2-6.

Polya, G. (1973). How to solve it. Princeton, N.J.: Princeton University Press.

Reitman, W. R. (1965). Cognition and thought. New York: Wiley.

Resnick, L. B. (1987). 'Learning in and out of school', *Educational Researcher, 16,* 13-20.

Reys, R. E., Lindquist, M. M., Lambdin, D. V., Smith, N. L., & Suydam, M. N. (2004). *Helping children learn mathematics* (7th ed.). Hoboken, NJ: John Wiley & Sons.

Schoenfeld, A. H. (1985). *Mathematical problem solving.* Orlando, FL: Academic Press.

Schoenfeld, A. H. (1987). Cognitive science and mathematics education: An overview. In A. H. Schoenfeld (Ed.), Cognitive science and mathematics education (n.p.). Hillsdale, NJ: Lawrence Erlbaum.

Steffee, L. P., (1994). Children's multiplying scheme. In G. Harel and J. Confrey (Eds.), The Development of multiplicative reasoning in the learning of Mathematics. (pp. 3-39). New York: State University of New York Press.

Wheatley, G. (1991). Constructivist perspectives on mathematics and science learning. *Science Education, 75*(1), 9-21.