Unpacking First Year University Students' Mathematical Content Knowledge Through Problem Solving

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ABSTRACT

Current calls for reform of education include suggested changes in the mathematics content courses required for university education programs. One suggestion is the inclusion of more complex problem solving activities. The context of this study was a homogeneous group of first year college students majoring in engineering. The researchers investigated the use of problem solving as a vehicle for these students' to: unpack previously learned mathematics; assess understanding; reconstruct understandings; and connect mathematical concepts for deeper understanding. The researchers considered what students bring with them to the college mathematics content classroom based on their national examination grades (SPM). They found that the students have an instrumental understanding rather than a relational understanding (Skemp, 1976), where they were not able to unpack their mathematical content knowledge and apply it to new contextual situations. The study adds to the accumulation of evidence regarding the nature of students' understanding of basic mathematical concepts and some critical factors to be taken into account in facilitating their development. Secondly, the grades obtained in the national examination of Sijil Pelajaran Malaysia (SPM) for mathematics did not indicate their mathematical knowledge in problem solving. This study also indicates that there was no difference in about 50% of

the test items performance between "A Math students" and "Non A Math Student" on the problem solving test. In summary, this study seems to indicate that that majority of this college freshmen have learnt how to do numerical computation at the expense of learning how to think and to unpack their mathematical content knowledge.

Background of the Study

Students from secondary schools bring a store of "learned" mathematics content knowledge with them to the college mathematics classroom. Research has shown that for some it consists of bits and pieces of disconnected information learned by rote, and it is often not useful or coherent (Featherstone, Smith, Beasley, Corbin, & Shank, 1995). Frequently, this mathematical content knowledge is tacitly rather than explicitly understood. Students can go through the mathematical motions to successfully complete exercises, but may not understand what they are doing, why it works, when it is appropriate to use, or if their answer makes sense (Ball, 1991, 1998).

Students coming to college need to unpack their mathematical knowledge which they bring from school to allow them to examine the undergirdings and interconnections of college mathematics with other relevant areas of mathematical application such as in physics. The examination by the students of these areas of content knowledge such as algorithms, definitions, and properties, will enable them to assess and identify this knowledge and their application and understanding. It should reveal mistakes or misconceptions. According to Ball (2002) in her study, this unpacking also serves as a conduit for preservice teachers to begin developing a personal relationship with mathematics which the researcher believes is very applicable to college students as well. From the constructivist perspective, learning is a process whereby learners actively construct their own understanding rather then passively absorb or copy the understanding of others (Cobb, Wood & Yackel, 1991; Steffee, Cobb & von Glasersfeld, 1988; von Glasersfeld, 1987). In other words, it assumes that concepts are not taken directly from experience, but that a person's ability to learn from and what is learnt from an experience depends on the quality of the ideas that the person is able to bring to that experience.

Researchers such as Skemp (1976, 1977, 1979, 1986, 1989) have uncovered levels of understanding of content. He differentiated between instrumental and relational understanding of mathematics. Instrumental understanding involved knowing what to do to get an answer whereas relational understanding involved knowing the what and the why. Relational understanding is sometimes understood to mean conceptual understanding. It involves a greater cognitive connectivity of the mathematical knowledge. The development of instrumental understanding became associated with habit training although this may not always have been the original intention of the teacher. The "issue of instrumental learning is rather more complex than the issue of learning by rote, for teachers do try, wherever possible, to teach algorithms for relational understanding. Pupils, however, often only retain the procedure and not the meaning" (Orton, 1989: 165)

Problem Solving

When we trace the history of problem solving in school mathematics, we find that it has often meant solving highly structured word problems appearing in texts. Often, the word problems were developed by the teacher (authority) to provide practice for prescribed computational procedures rather than to encourage problem solving in a broader sense. Steffe (1994) states that:

> *The current notion of school mathematics is based almost exclusively on formal mathematics procedures and concepts that, of this nature, are very remote from the conceptual world of the children who are to learn them (p. 5)*

A study by Parmjit (1998) found that only a small percentage of students who did well in the Penilaian Menengah Rendah (PMR) were able to solve complex proportional problems and the grades obtained in this exam were not indicative of their knowledge of ratio and proportion. He noted that:

> *the more we focus on raising test scores, the more instruction is distorted and the less credible are the scores themselves. Rather than serving as accurate indicators of students knowledge and performance, the tests become indicators of the amount of instructional time and attention paid to the narrow range of skills assessed (p. 107)*

In fact, basic computation skills have been the focus for competency tests through the years, spawning textbooks and instructional emphases aimed at developing these skills. Students have learned how to do numerical computations at the expense of learning how to think and solve problems. If one has ready access to a solution scheme for a mathematical task, that task is an exercise and not a problem. A *problem* is a question that exercises the *mind.* We should be cautioned against claiming to emphasize problem solving when in fact the emphasis is on routine exercises and procedural knowledge. From various studies involving problem solving instruction, Suydam (1987) concluded that:

> *If problem solving is treated as"apply the procedure," then the students try to follow the rules in subsequent problems. If you teach problem solving as an approach, where you must think and can apply anything that works, then students are likely to be less rigid. (p. 104)*

An emphasis on problem solving as a learning tool must take place, for today's students are the people who will lead the world in the future. The tools they learn in school now will be applied to the many serious problems they will face in the "real world". In view of this, research in problem solving has a special importance in the study of mathematics.

Significance and Purpose of the Study

The purpose of this study is to investigate college freshmans' understanding and ability to unpack their mathematical knowledge through problem solving. Successful mathematics students do indeed construct a fairly large number and a variety of algorithms in order to continue to achieve good results in mathematics examinations. However, it of interest to determine the quality of this knowledge as the quality of students' mathematics knowledge is always a crucial matter. The single most important factor determining the quality of knowing is the quality of the students' experiences in constructing their knowledge.

This study will seek to provide insight into whether these students have good reasoning abilities or are just performing algorithmic procedures without making any deep sense of why these procedures worked. It could be useful for curriculum developers and educators, especially in Malaysia, in evaluating strategies and methods used in the teaching and learning of mathematics in universities. Furthermore, the importance of problem solving, coupled with difficulties in acquiring the skill gives a rationale for studying problem solving in mathematics learning.

Objectives of the Study

This study aims to develop a comprehensive description of college freshman's thinking and reasoning capabilities in solving non-routine problems in a quantitative and qualitative manner. While the qualitative will The quantitative study aims to examine college freshman's performance in problem solving situations. Specifically, the questions addressed are:

1. What are the levels of college freshman's understanding in solving non-routine problems?
2. Do college freshmen who have obtained a high grade in national examination (SPM) mathematics also score likewise on the problem-solving test?
3. Are there differences among the mean scores of "A math students" and "non A math students' " achievement in the problem-solving test?
4. What difficulties do college freshmen encounter in solving non-routine problems?

Design of the Study

The methodology used in this study was both qualitative and quantitative in nature. Qualitative data from clinical interviews with college freshmen gave the researchers an in-depth understanding of these students' heuristic actions, exploration of the mathematical processes, and tacit mathematical understanding that constitute thinking mathematically in problem solving. The written assessment provided both quantitative and qualitative data about these students' relational understanding of their application of mathematical concepts in problem solving.

This study was conducted during the May/June semester at a college in Malaysia. Five hundred thirty six college freshmen, ages 18-19, participated in this study. The rationale for choosing a sample of college freshmen was that students at this level have been formally taught the basic mathematical concepts needed to solve problems during their high school career. This enabled the researchers to evaluate the students ability to unpack their knowledge, that is, to investigate if college freshmen could apply their knowledge in solving problems that were new but within their zone of potential construction.

Secondly, these college students have been through a standardized examination, namely the SPM, and thus the researchers could easily cluster the students based on their SPM results in mathematics and consequently pursue the issue of how well SPM mathematics grades reflect the students' relational understanding of mathematics.

The assessment for this study comprised an instrument developed by the researchers. The purpose of the instrument was to assess students' relational understanding of mathematics. Its main goal was to provide key information concerning students' functioning in the area of problem solving, especially their ability to cope with different problem contexts.

There were all together 12 items in the test and the language utilized for the items was in the native language of the students, Bahasa Melayu. This was undertaken to avoid any language reactive effects. The responses for the items were grouped into categories according to the criterion behaviour exhibited. A numerical value was assigned to each of these criterion behaviours. Students' responses were categorized on a 4 point scale based on the reasoning employed. The 4-point scale used was:

4. All correct, good reasoning
3. Good reasoning, small error(s)
2. Some promising work but it is not clear on whether a solution would be reached
1. Some work but unlikely to lead to a solution
0. Blank

Table 1 reveals that 345 students obtained a grade 1A, 108 a grade 2A, and the rest obtained grades 3B, 4B, 5C and 6C in their SPM mathematics examination. In other words, 453 subjects obtained distinctions (1A and 2A) as compared to 83 who did not.

Table 1: Distribution of Subjects According to SPM Mathematics Grades

Grade	Frequency	Percentage
1A	345	64.4
2A	108	20.1
3B	53	9.9
4B	16	3.0
5C	7	1.3
6C	7	1.3
Total	536	100.0

For this study, the 453 students who obtained 1A and 2A grades in SPM mathematics were clustered as "A Math Students" compared to the balance of 83 as "Non A Math Students". The distribution of these two groups of students is shown in Table 2. It is not surprising to see a large number of "A math" students in this study because one of the pre-requisites for entering college is to have a minimum grade of 6C in SPM mathematics.

Table 2: Clustering Students by SPM Mathematics Grades

Type of Student	Frequency	Percentage
A Math	453	84.5
Non A Math	83	15.5
Total	536	100.0

Results of the Study

The results of the study that follows detailed an item analysis followed by an analysis for each item individually before an overall assessment is made.

Item Analysis of Test Items

An item analysis of the twelve test items in the instrument was conducted. Table 3 shows the average score and the standard deviation for each of the twelve test items attempted in the instrument. The maximum score for each item is 4.

Table 3: Students' Performance on Test Items

Item Number	Mean Score	Standard Deviation
Item 1	1.76	0.87
Item 2	2.69	1.38
Item 3	2.25	1.19
Item 4	2.59	1.46
Item 5	2.22	1.38
Item 6	1.85	1.00
Item 7	1.80	1.23
Item 8	2.03	0.80
Item 9	1.33	1.00
Item 10	1.21	0.99
Item 11	2.44	1.21
Item 12	2.46	1.41

The items that have very low mean scores (less than 2.00) are Item 1, 6, 7, 9, and 10. Those with higher scores (more that 2.50) are Item 2 and 4. The above data can be an initial indication of college freshman's fundamental relational understanding of mathematical concepts in problem solving. Confirmation was sought through interviews.

Item 1

The cost of a lunch of 3 sandwiches, 7 cups of coffee and 1 donut is $3.15. The cost of a lunch of 4 sandwiches, 10 cups of coffee and 1 donut was $4.20 at the same cafe. How much will 1 sandwich, 1 cup of coffee and 1 donut cost?

In this item, a majority of students were able to write the two equations, $3s + 7c + 1d = 3.15$ and $4s + 10c + 1d = 4.20$, where s, c and d represent 'sandwich', 'coffee' and 'donut' respectively. A majority of the students could classify the problem as a system of linear equations with 2 equations and 3 unknowns. They solved the equations simultaneously but stopped when they obtained an equation with 2 unknowns. From the interviews, it was found that many of the students had difficulties in solving the problem since their usual expectation is to find the cost for each food rather than a set of relationships.

A low mean of 1.76 in this item indicates that the students were just able to write the two linear equations but could not proceed from there although this fundamental understanding of simultaneous equation has been taught in schools since form 3. From the cross-tabulation data in Table 5, it is interesting to note that approximately 90.7% of the "A Math" students were not able to solve Item 1. Approximately 8.4% of the students got the correct answer for this item which is actually quite a dismal result. Perhaps the unrealistic prices contributed to the difficulty of the problem.

Item 2

Pele and Maradona were great soccer players, Pele scored 300 goals in 400 matches, while Maradona scored 400 goals in 500 matches. Who had a better scoring record, Pele, Maradona or did they have the same scoring record? (Please explain).

This item was constructed by Parmjit (1998) in which the fundamental concepts involved were proportion and ratio. The observed heuristic actions of the students' responses were as follows:

- Both missed 100 goals, so the scoring record were the same
- Some of them computed the ratio (400/300 for Pele and 500/400 for Maradona) and obtained 1.33 and 1.25 respectively. Then they reasoned that since the value of the former is greater than the latter, with the conclusion of Pele has a better scoring record. Surprisingly, they did not seem to be able to articulate what the number represented.

Table 5 shows that 53.4% of the students gave the correct answer on this item. The mean test score is 2.69. From the cross-tabulation data, it is observed that 56.3% of "A Math" students were able to solve Item 2. Only 43.7% of them were unable to do so. This is an encouraging sign, more consistent with the achievement level represented by their mathematics grades.

Item 3

A dog chasing a rabbit, which has a start of 45 m, jumps 3 m every time the rabbit jumps 2 m. In how many leaps does the dog overtake the rabbit?

This item had a mean of 2.25 with approximately 39.6% giving a correct response. This is an algebraic task and some students were observed using interesting heuristic actions to solve the problem such as:

> *Difference of distance of every leap is 1 m. To cover the difference of 45 m, it re*quires *45 leaps. Therefore, the dog will overtake the rabbit in the 46th leap.*

A majority of them saw the problem as a difference of 1 meter between each jump of the rabbit and dog. Then they classified the problem as sequences:

2, 4, 6, 8, ... as the sequence for the distance traveled by the rabbit and

3, 6, 9, 12, ... as the sequence for the distance traveled by the dog.

Surprisingly, students faced difficulty in expressing the problem as a mathematical algebraic equation of $45 + 2(x) = 3x$ where x is the number of jumps. None of the students used this algebraic equation to solve the problems, as one would expect from college students.

Table 5 summarizes the cross tabulation data for Item 3, with only 39.6% of the sample managing to exhibit a good to excellent reasoning while approximately 57.6% of the “A Math” students were not able to solve Item 3.

Item 4

Find the value of 1 - 2 + 3 - 4 + 5 - 6 + ... - 100.

A majority of students tried to classify the sequences as a geometric series or arithmetic series by finding the common ratio or the common difference, which led to a wrong answer, that did not make sense. They did not realize that this series was neither a geometric nor an arithmetic progression.

However, the majority of the 51.5% of students who got this item correct, separated this series as sum of 1 + 3 + 5 + 7 + … and -2 + - 4 + -6 … . After finding the sum of each series, they added the sum to get the answer.

Table 5 shows the cross tabulation data for Item 4. The mean score is 2.59 with approximately 51.5% (276) of students able to solve the problem. The encouraging sign is that more than half of the “A Math” students are among the 51.5%. However, there were 46.8% of “A Math” students who were not able to achieve a high level of reasoning in solving the problem at hand.

Item 5

This year in mathematics the teacher will be giving 5 tests, each scored out of 100. If Halimah wishes to get an A+ average (90 or above), what is the lowest possible score she can receive on a test?

Approximately 60% of the students gave the answer as 90, by looking at the average from 450 marks divided by 5 tests. Basically, they did not make any assumptions on how Halimah did on the other tests. Those who did make assumptions didn’t assume 100 to be the marks for the other tests.

Quantitative data gave a mean score of 2.22 with approximately 35.4% of the sample getting it correct. Table 5 gives the cross tabulation data for Item 5. There seems to be a large number (61.1%) of "A Math" students who were not able to solve Item 5. That means only 38.9% of "A Math" students were able to solve this item.

Item 6

A scientist with metric mania has designed a metric clock with 10 hours in a day and 100 minutes in an hour. When the metric clock reads 5 o'clock, it is really noon (12.00). What is the real time when the metric clock reads 6:75?

Analysis of the responses for Item 6 indicated that a majority of students who obtained the difference of one hour and seventy five minutes did not know how to change it to real time. In other words, they took the 1 hour 75 minutes as 175 minutes on an ordinary clock and ended up with the answer as 2.55 pm. Some even took the ratio of 10 to 12 instead of 10 to 24 to solve this problem.

A low mean of 1.85 meant only 15.3 % of the students got it correct for this item. Table 5 shows that only 16.3% of the "A Math" students were able to solve the item. That means 83.7% or more that four-fifths of them were not able to solve this item.

Item 7

Eva and Alex want to paint the door of their garage. They first mix 2 cans of white paint and 3 cans of black paint to get a particular shade of gray. They add one more can of each. Will the new shade of gray be lighter, darker or are they the same?

Approximately 76.4% of the students got this item wrong with a low mean of 1.80. They reasoned that if one adds 1 can of black paint and 1 can of white paint, the colour should still be the same. Their reasoning relied on the assumption that since one can of each paint were added and the quantity of both colours were the same, the outcome of the new mixture should have the same shade. Their reasoning was based on the primitive additive reasoning and approximately 42.2% of the students gave this additive reasoning. In short, these students failed to construct a coordination of two ratios simultaneously as: 2 white to 3 black and 3 white to 4 black. We can also compare the white ratios of 2/5 with 3/7 with greater the white ratio, the lighter the shade it is.

Analysis of Table 5 indicates that only a small number of students managed to solve Item 7, although a big portion of them were "A Math" students. Approximately 74.8% of the "A Math" students were unable to solve this item.

Item 8

If it takes 9 workers 5 hours to mow a certain lawn, how long would it take 6 workers to mow the same lawn? (Assuming that the workers are all performing at the same rate and all working for the entire time).

In this item, 78.7% of the students failed to see an inverse proportion relationship and solved the question by utilizing a cross multiplicative structure. Many of them used the following heuristics:

9 workers = 5 hours
6 workers = X
X/5 = 6/9; 9X = 30; x = 30/9 = 3 1/3 hours.

In fact, 57.8% of the students utilized mechanical reasoning as shown above. Here, they did not reason what each number represents and what they were actually computing. Logically, they should have realized that the answer they produced (3 1/3 hours) implied that more people take a longer time to finish the job.

The mean score for this item was 2.03. Table 5 shows the majority of the students exhibited poor reasoning which unable them to solve this item. More shockingly, approximately 85.0% (385) of the "A Math" students were unable to solve it.

Item 9

An old antique bicycle has wheels of unequal size. The front wheel has a circumference of 8 feet. The back wheel has a circumference of 10 feet. How far has the bicycle gone when the front wheel has turned 20 more revolutions than the back wheel?

Observations of students' responses to this item indicate that approximately 90% of the students could not even get to a stage to show any promising heuristic reasoning. They faced great difficulties in expressing the problem as a mathematical expression. The interviews revealed that a majority of the students did not realize that both tyres traveled the same distance.

The mean score for this item was 1.33 with only 8.4% getting it correct. Table 5 shows that 91.6% of the students were unable to solve this item. It is disappointing that approximately 90.1% of "A Math" students did not show good reasoning abilities to solve the item.

Item 10

A printer uses 999 digits to number the pages of a book. How many numbered pages are there in the book?

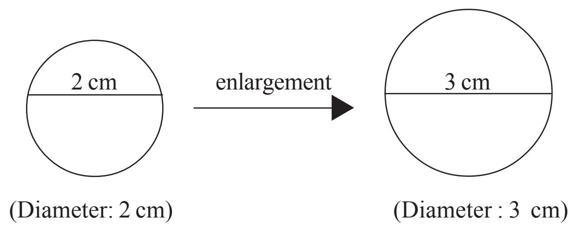
A majority of the students obtained the answer as 999 pages. This was because they reasoned one page per digit. For example, they used 11 or 444 as one digit instead of as two and three digits respectively. From the interview, it was obvious that the students did not understand the meaning of the word "digit".

Thus students took the meaning of a digit as that of a term. So "999 digits'' means 999 terms which led to the answer 999 pages. The students were not able to see that the number 10 contains 2 digits which are 1 and 0.

The mean score for this item was 1.21 with only 11.0% who got it correct. Additionally, 87.9% of the "A Math" students were unable to exhibit good reasoning abilities in solving this item.

Item 11

A copy machine can enlarge any figure. Below is an example.



When I copy a rectangle which is 5 cm by 7 cm, what will the area of the new rectangle be after it is enlarged, using the same copy machine setting?

In this item, approximately 24% of the students utilized the additive reasoning in trying to solve this problem. A majority of the students gave the following reasoning in obtaining their answer … "2 + 1 = 3, so the new dimension will be (5 + 1) by (7 + 1) that is 6 cm by 8 cm. So the area will be $48cm^2$." They were unable to see how enlargement works with problems that involve equal proportions.

The mean score for this item was 2.44 with 39.5% of the students getting it correct. However, there are still 57.8% (262) of the "A Math" students who were not able to solve this item.

Item 12

The ages of ten members of the Girl's Club range from 4 to 13, and each girl is of a different age. Two girls from each of five families belong to the club. The sums of the ages of each pair of sisters are 10, 13, 17, 22, and 23. One girl is 7; how old is her sister?

Approximately 53% of the students were able to find the possible pairs. However, a number of them did not note that the sisters ages should be in the range of 4 to 13 years.

The mean score for this item was 2.46 with 52.8% getting it correct. Table 5 shows that 55.0% (249) of the "A Math" students were able to exhibit good to excellent reasoning in solving this item, which is an encouraging result.

Comparison of Problem Solving Ability of "A Math" Students and "Non A Math" Students

Table 4 shows that the mean score for both the "A-Math" and "Non A Math" students receive is a low 26.1 and 21.9 respectively with a maximum score of 48. In table 5, it details the item analysis for each group according to percentages of correct responses. The percentage of correct responses for items 1, 6, 8, 9, and 10 was less than 15% whereas items 2, 4 and 12 was about 50% correct. It shows that even A-students who got an A in their national examination faced difficulties in these items. It is quite surprising that most "A Math" students could not solve some of these problems, especially the items which showed quite a low mean score. Those items are Item 1 which requires the ability to construct and solve simultaneous equations, Item 6 which requires the skill of interpreting ratio problem and Item 10 which requires some skills in interpreting and organizing the data.

Table 4: Mean and Standard Deviation of "A Math" and "Non A Math" Students

Type of Student	Mean (Max. 48)	Std Dev
A Math	26.1478	6.70852
Non A Math	21.9162	5.95008

Table 5: Item Number with Correct Percentage between "A Math" and "Non A Math" Students

Item #	% Correct		
	"A Math" Students	"Non A Math" Students	Overall
1	9.3	3.6	8.4
2	56.3	37.3	53.4
3	42.4	24.1	39.6
4	53.2	42.2	51.5
5	38.9	16.9	35.4
6	16.3	9.6	15.3
7	25.2	15.7	23.7
8	12.8	3.6	11.4
9	9.9	0	8.4
10	12.1	4.8	11.0
11	42.2	25.3	39.6
12	55.0	41.0	52.8

The cross tabulation of each item in Table 6 indicates that there may be some interesting relationships between the proportions of different types of responses from the two groups of college freshmen. Indeed, when the difference of mean scores for the "A Math" and "Non A Math" students for each item was calculated, there were significant differences between the two groups of students on several items. There was a significant difference at the 0.05 level between the means of the "A Math" students and the means of "Non A Math" students for item 2, item3, item 5, item 7, item 9, item 11 and item 12, while in Item 1, Item 4, Item 6, Item 8, Item 10, there was no significant difference between the means of the two groups of students. In summary we can conclude that there was no difference in about 50% of the items between "A Math" students and "Non A Math" students in the problem-solving test.

Table 6: Performance of "A Math" Students Versus "Non A Math" Students

Item	Mean of "A Math" Students	Means of "Non A-Math" Students	Mean Difference	Sig.
1	1.78	1.61	.17	.106
2	2.78	2.20	.57*	.000
3	2.32	1.87	.45*	.001
4	2.64	2.30	.34	.051
5	2.32	1.67	.64*	.000
6	1.89	1.69	.20	.098
7	1.85	1.53	.32*	.031
8	2.06	1.90	.15	.107
9	1.37	1.13	.24*	.047
10	1.22	1.17	.05	.673
11	2.50	2.12	.38*	.008
12	2.52	2.18	.24*	.046

*The mean difference is significant at the .05 level.

Conclusion and Discussion

The data from this research identifies students' inability to unpack the subject knowledge in mathematics as being a contributory factor in low standards of mathematics attainment of college students in the problem solving test. It seems that students have learnt how to do numerical computation as a procedure at the expense of learning how to think and solve problems. Specifically, the following general conclusions were obtained from analyses of the quantitative and qualitative data.

1. Students were not able to unpack their mathematical knowledge of problem solving items.
2. Students obtained a very low overall mean score in the problem solving test.
3. The Mathematics grades obtained in the Sijil Pelajaran Malaysia do not correlate with a student's mathematical knowledge on the problem solving test.

4. Students who obtained a high grade on the SPM Mathematics examination did not score likewise on the problem solving test.
5. From the interviews, it was found that the primary goal for most of these students is to find an algorithm that will give them the answer quickly. Many students know how to carry out basic mathematical procedures when problems are presented in symbolic form but are not able to apply these procedures to solve problems presented in words. In short, this study suggests that the difficulty of students lies in understanding the problems, rather than executing procedures. In Skemp's (1989) language, the students have concentrated upon developing an instrumental understanding of their mathematics in secondary school.
6. Students exhibit additive reasoning most of the time in proportion tasks, even in inappropriate situations. Many do not seem to have an overall view, nor do they know what is represented by the numbers obtained.

Results from this research on students understanding in problem solving indicate that certain incorrect ideas about the fundamental mathematical concepts are common to the college freshmen. There is evidence that college freshmen often have many of the same conceptual and reasoning difficulties that are common among younger children. There is often little change in conceptual understanding before and after formal instruction. Moreover, students are often unable to apply the concepts that they have learnt to the task of solving non-routine quantitative problems.

The quantitative data obtained from posing these problems seem to show that for most students in this study, their mathematics instruction was procedural without sense-making: one learns to read the problem, extract the relevant numbers and the operation to be used, to perform the operation, and to write down the result – without even thinking about what it all means. Utilising the cross multiplication approach in solving these proportion problems simply becomes an act of symbolic manipulation without requiring that individuals make sense of what they are doing. This clearly shows students' memorising procedures and rules in applying the cross-multiplication procedure rather than emphasising on making sense of the outcome of such procedures. This cross-multiplication algorithm is an efficient way of getting answers but is often used without meaning. In order for students to develop structural understanding, we should perhaps emphasize on giving them experiences that can create a solid foundation for these concepts (Kieran, 1994).

Relationship between Students' Mathematics Grades in SPM Examination and their Performance in the Problem Solving Test

The outcome of this study indicates that there was no difference in 50% of the items in the problem Solving test between "A Math" students and "Non A Math" students. One would expect these college freshmen, especially those with A's from the SPM mathematics paper, to be excellent problem solvers. But sad to say, this is not so. The performance of the A students was very disappointing. The question, which arises here, is: "How well do the current national examination grades reflect the mathematical knowledge of students?"

The grades obtained by students on the SPM do not indicate their mathematical knowledge. These grades do not measure how well educated the students are. As Parmjit (1998) observed: "the more we focus on raising test scores, the more instruction is distorted, and the less credible are the scores themselves" (p. 107). Rather than serving as accurate indicators of student's knowledge and performance, the tests become indicators of the amount of instruction time and attention paid to a narrow range of skills assessed. Furthermore as Kieran (1988) has pointed out, symbolic knowledge not based on understanding is highly dependent on memory and is subject to deterioration. If schools place too much emphasis on the procedures rather than the process of learning. So, when students "practice" these problems, they are practicing to get the correct answer. In other words, they ignore things like context, structure and situations, and students do not have the occasion to generate the "richly inter-connected spaces" that Cooper (1988) has identified as being crucial for constructing mathematical knowledge. They end up with islands of superficial knowledge without a canoe to get from one end to the other.

To Polya (1973), problem solving was a major theme for doing mathematics and "teaching students to think" was of primary importance. "How to think" is a theme that underlies much of genuine inquiry and problem solving in mathematics. However, care must be taken so that efforts to teach students "how to think" in mathematics problem solving do not get transformed into teaching "what to think" or "what to do". This is, in particular, a by-product of an emphasis on procedural knowledge about problem solving as seen in the linear framework of Malaysian textbooks.

From the interviews with students, the joy of doing these kinds of problems allows the students to experience a range of reflection associated with various stages in the solution process. They also show the willingness and desire to engage with the tasks for a longer period of time. And when students are challenged to articulate their own ideas, they learn about their own powers of thought (Schifter & Fosnot, 1993). Students' developing ability to reflect on their own thinking and actions is evidence of the emerging nature of their mathematical understanding (Carpenter & Lehrer, 1999).

Although it is this engagement that initially motivates the solver to pursue a problem, it is still necessary for certain techniques to be available for the involvement to continue successfully. Hence more needs to be understood about what these techniques are and how they can best be made available. Problem solving has a place in our curriculum as spelt out in the Malaysian mathematics syllabus, however, it is often used in a token way as a starting point to obtain a single correct answer, usually by following a set of "correct procedures".

In summary, the findings of this study have potential significance for the teaching and learning of mathematics in universities and school in general. It supports the changing of our perspective on mathematics learning by making problem solving the focus of school mathematics. Problem solving is encouraged for the need to accomplish the instructional goals of learning basic facts, concepts, and procedures, as well as goals for problem solving within problem contexts. In other words, mathematics instruction should be designed so that students experience mathematics as problem solving in schools in order for them to be able to unpack this knowledge at the university.

Reduce the emphasis on drill of basic mathematical procedures in teaching students to solve problems. The researchers contention here is that there should be a balance between problem solving and basic skills in mathematics because it is acknowledged that students cannot be successful problem solver without a solid foundation in basic skills. The national curriculum has been emphasizing too much on the latter and de-emphasizing the former. The result is to increase students' instrumental understanding in mathematics. There seems to be a mismatch between the learning in high schools and unpacking needed in university for mathematics learning. It is important not to lose sight of the intimate relationship between basic skills and the acquisition of problem solving skills. Educators needs to be aware of the knowledge and understanding of mathematics that students need in order to underpin effective teaching of mathematics at tertiary level. College students need to unpack and revisit the mathematics they have previously learned from high school. The use of problem solving in the college mathematics content classroom enables this unpacking and results in deeper understanding and a change in mathematical understanding and beliefs.

References

Ball, D. L. (1991). Research on teaching mathematics; Making subject matter knowledge part of the equation. In J. Brophy (Ed.), *Advances in Research on Teaching: Teachers' Subject Matter Knowledge and Classroom Instruction*, (Vol. 2). Greenwich, CT: JAI Press.

Ball, D. L. (2002, January). *Teaching future teachers workshop*, San Diego, CA, January 9-14.

Charles, R. I., Lester, F. K., & O' Daffer, P. (1987). How to evaluate progress in problem solving. Reston, VA: National Council of Teachers of Mathematics.

Cobb, P., Wood, T. and Yackel, E. (1991). A constructivist approach to second grade mathematics. In von Glaserfield, E. (Ed.), *Radical constructivism in mathematics education* (pp. 157-176).

Dordrecht, The Netherlands: Kluwer Academic Publishers. Cockcroft, W.H. (Ed.) (1982). *Mathematics counts.* Report of the committee of inquiry into the teaching of mathematics in schools, London: Her Majesty's Stationery Office.

Dauw, D. C (1976). *Creativity and innovation in organization*. Dubuque, Iowa.

Dewey, J. (1991). How we think, in Judith J. Lambrecht (1999), *Teaching problem solving for employment preparation*, http:// wcfe. coled. umn.edu /jlambrec/ BIE5401 / Fall1999/prob2.htm. Accessed on 14 November 2004.

Featherstone, H., Smith, S. P., Beasley, K., Corbin, D., & Shank, C. (1995). *Expanding the equation: Learning mathematics through teaching in new ways*.

(Research Report 95-1). East Lansing: Michigan State University, National Center for Research on Teacher Learning.

Garfola, J. & Lester, F. K. (1985). Metacognition, cognitive monitoring, and mathematical performance. *Journal for Research in Mathematics Education, 16*, 163-176.

Henderson, K. B. & Pingry, R. E. (1953). Problem solving in mathematics. In H. F. Fehr (Ed.), *The learning of mathematics: Its theory and practice* (pp. 228 – 270). Washington, DC: National Council of Teachers of Mathematics.

Kantowski, M. G. (1977). Processes involved in mathematical problem solving. *Journal for Research in Mathematics Education, 8,* 163-180.

Kieran, T. (1988). Personal knowledge of rational numbers: Its intuitive and formal development. In J. Hiebert & M. Behr (Eds.), *Acquisition of mathematics concepts and processes* (pp. 45 – 90). New York: Academic Press.

Kieran, T. (1994). Multiple views of multiplicative structure. In G. Harel & J. Confrey (Eds.), *The development of multiplicative reasoning in the learning of mathematics* (pp. 387 – 397). State University of New York Press.

Lesh, R. (1981). Applied mathematical problem solving. *Educational Studies in Mathematics, 12*(2), 235-265.

Mathematics Textbook (1991). *Matematik Tambahan Tingkatan 4 dan 5* Kurikulum Bersepadu Sekolah Menengah. Zohriah Trading.

McAllister, C. Howard (1999). Problem solving and learning. In http://www. hawaii. edu /suremath/home1.html. Accessed on 21 November 2004.

National Council of Teachers of Mathematics (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA.

Newell, A. & Simon, H. A. (1972). *Human problem solving*. Englewood Cliffs, NJ: Prentice Hall.

Parmjit S. (1998). Understanding the concepts of proportion and ratio among students in Malaysia.Unpublished doctoral dissertation. Florida State University.

Perkins, D. N. (1981). *The mind's best work.* Cambridge, MA: Harvard University Press.

Polya, G. (1973). *How to solve it*. Princeton, N.J.: Princeton University Press.

Polya, G. (1962). *Mathematical discovery: On understanding, learning and teaching problem solving*. New York: Wiley.

Reitman, W. R. (1965). *Cognition and thought*. New York: Wiley.

Resnick, L. B. (1987). Learning in and out of school, *Educational Researcher, 16,*13 – 20.

Schifter, D., & Fosnot, C. (1993). *Reconstructing mathematics education*. NY: Teacher College Press.

Schoenfeld, A. H. (1987). Cognitive science and mathematics education: An overview. In A. H. Schoenfeld (eds.), *Cognitive science and mathematics education* (pp.1-31). Hillsdale, NJ: Lawrence Erlbaum.

Schoenfeld, A. H. (1979). Can heuristic be taught? In J. Lockhead (ed.), *Cognitive process instruction* (pp. 315-338). Philadelphia, PA: Franklin Institute Press.

Schoenfeld, A. H. (1985). *Mathematical problem solving*. Orlando, FL: Academic Press.

Schoenfeld, A. H. (1988). When good teaching leads to bad results: The disaster of "well taught" mathematics classes. *Educational Psychologist, 23,* 145 – 166.

Shaffer, P. S. & McDermott, L. C. (1992). Research as a guide for curriculum development: An example from introductory electricity. Part II: Design of an instructional strategy. *American Journal of Physics, 60*, 1003 – 1013.

Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics teaching*, *77*, 1 – 7.

Skemp, R. (1977). Professor Richard Skemp, interview by Michael Worboys. *Mathematics in School, 6* (2), 14-15.

Skemp, R. (1979). *Intelligence, learning and action*. Chichester: Wiley & Sons.

Skemp, R. (1986). *The psychology of learning mathematics (2nd ed).* London: Penguin.

Skemp, R. (1989). *Mathematics in the primary school.* London: Routledge.

Smith, J. P. (1974). The effects of general versus specific heuristics in mathematical problem-solving tasks (Columbia University, 1973). Dissertation Abstracts International, 34, 2400A.

Stanic, G., & Kilpatrick, J. (1988). Historical perspectives on problem solving in the Mathematics curriculum. In R. I. Charles & E. A. Silver (Eds.), *The teaching and assessing of mathematical problem solving* (pp. 1 – 22). Reston, VA: National Council of Teachers of Mathematics.

Steffee, L. P. (1994). Children's multiplying scheme. In G. Harel and J. Confrey (eds.), *The development of multiplicative reasoning in the learning of mathematics* (pp. 3 – 39.). State University of New York Press.

Suydam, M. (1987). Indication from research on problem solving. In F. R. Curcio (Ed.), *Teaching and learning: A problem solving focus*. Reston, VA: National Council of Teachers of Mathematics.

Von Glaserfeld (1987). Learning as a constructive activity. In C. Janvier (Ed.), *Problem of representation in the teaching and learning of mathematics* (pp. 3 – 17). Hillsdale, NJ: Lawrence Erlbaum Associates.

Wheatley, G. (1991). Constructivist perspectives on mathematics and science learning. *Science Education, 75*(1), 9 – 21.

Wilson, J. W. (1967). Generality of heuristics as an instructional variable. Unpublished Doctoral Dissertation, Stanford University, San Jose, CA.

Woods, D., et al. (1997). Developing problem solving skills: The McMaster problem solving program. *Journal of Engineering Education, 86* (2), 75 – 91.